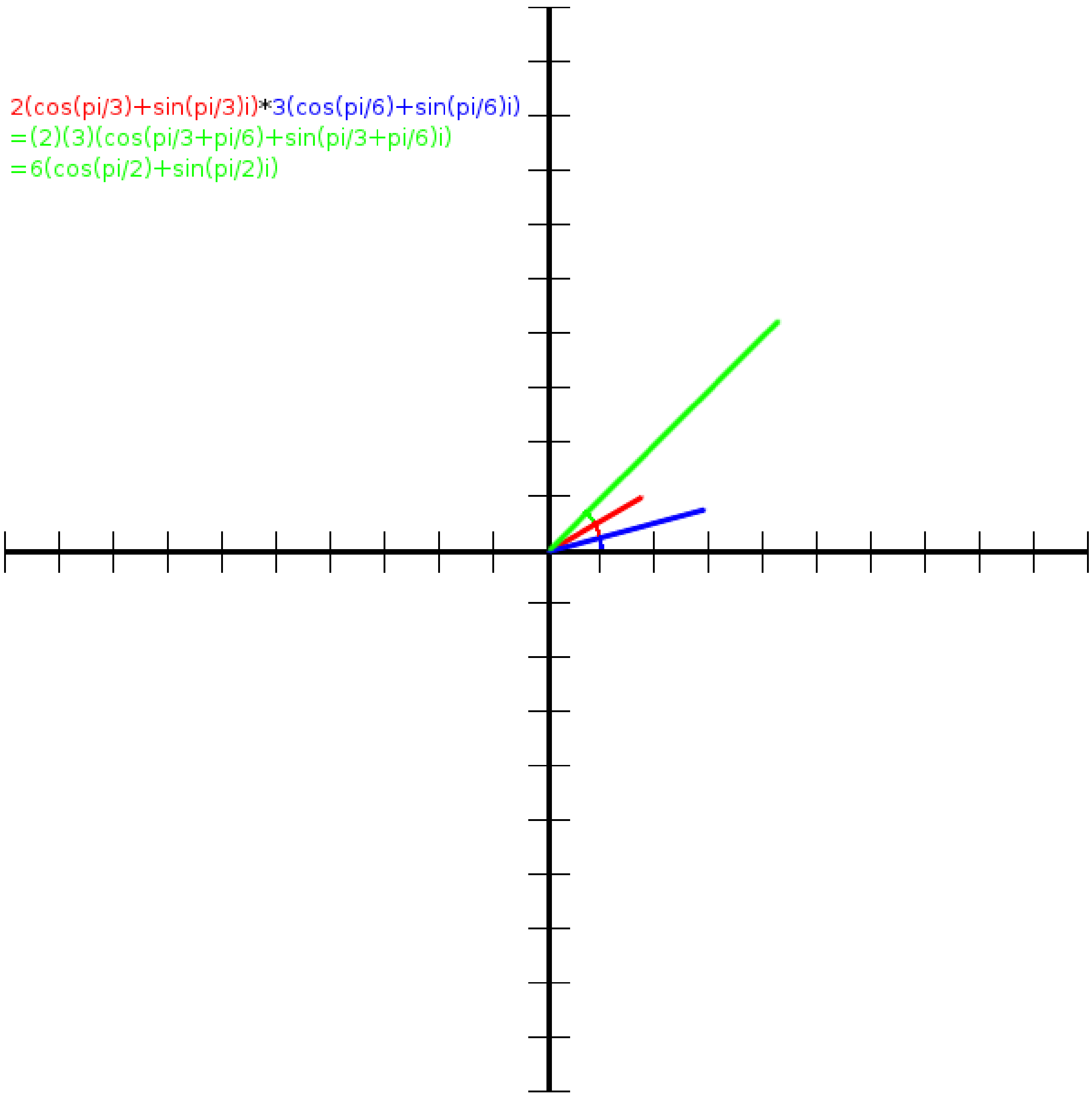

Hooray, a diagram with a scale and stuff! Opening up this lesson where we left off last time, this diagram shows multiplication of two complex numbers in polar form. In this case, we're looking at $2\left(\cos\left(\frac{\pi}{3}\right)+\sin\left(\frac{\pi}{3}\right)i\right)*3\left(\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{6}\right)i\right)=6\left(\cos\left(\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{2}\right)i\right)$. You can see how the angles of the red and blue lines can be added to get the angle of the green line. The magnitude of the green line is just equal to that of the red line times the blue line.

You'll also notice I started using radians – using radians will be really really important in a few lessons, as the equations will not work at all with degrees. If you don't know already, 360 degrees is 2pi radians. x radians means that the angle is equal to that of a vector that intersects with the unit circle at a point a distance of x along the unit circle away from the point (1,0).

Now then, from the syllabus we see that today's lesson is on real roots of complex numbers. We know that each nonnegative real number has 2 nth roots if n is even, and each negative number has 0 nth roots if n is even. If n is odd, any real number has exactly one nth root. On the other hand, any given complex number has n nth roots for any real n.

We can calculate exactly what those roots are by first putting some complex number into polar form:

$r(\cos(\theta)+\sin(\theta)i)$

And by using the property that $\sqrt[n]{x}\sqrt[n]{y}=\sqrt[n]{xy}$, we can reduce $\sqrt[n]{r(\cos(\theta)+\sin(\theta)i)}$ to: $\sqrt[n]{r}\sqrt[n]{(\cos(\theta)+\sin(\theta)i)}$. Ignoring $\sqrt[n]{r}$ for now, we know that $\sqrt[n]{(\cos(\theta)+\sin(\theta)i)}^n$ cancels out the radical. We also know from the last lesson that that would mean adding the argument of $\sqrt[n]{(\cos(\theta)+\sin(\theta)i)}^n$ to itself *n* times – multiplying the argument by *n*. So if we cancel the radical out by multiplying the argument by *n*, then we can derive the argument of $\sqrt[n]{(\cos(\theta)+\sin(\theta)i)}^n$ as θ/*n*.

So then:

$\sqrt[n]{r(\cos(\theta)+\sin(\theta)i)}=\sqrt[n]{r}(\cos(\frac{\theta}{n})+\sin(\frac{\theta}{n})i)$

Cosine and sine are also periodic functions, though. *cos(x) = cos(x+2pi)*, and *sin(x) = sin(x+2pi)*. Because of this, for any given complex number, we can add *2pi* to the argument and it will still be the same complex number. To find all the other roots of the complex number – each complex number has *n n*th roots for any real *n* – we must add *2pi* to the argument of *z* to get the next root of *z*, then add *4pi* to *z*, then *6pi*, *8pi*, continuing until the value is equal to that of the first root we found.

More formally:

$\{z:z^n=w\}=\{z:|z|^n=w\wedge Arg(z)\cdot n\equiv w\, mod\,(2\pi)\}$

With mod(2*pi) meaning Arg(z)*n plus some integer multiple of 2*pi is equal to w.

Optional exercises!

1. $\sqrt{4(\cos(\frac{3\pi}{4})+\sin(\frac{3\pi}{4})i)}$
2. $\sqrt{5(\cos(\frac{3\pi}{2})+\sin(\frac{3\pi}{2})i)}$
3. $\sqrt{i}$
4. $\sqrt[3]{8(\cos(\frac{7\pi}{4})+\sin(\frac{7\pi}{4})i)}$
5. $\sqrt[4]{9(\cos(\frac{8\pi}{5})+\sin(\frac{8\pi}{5})i)}$